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Modeling and optimizing a sub-centralized LED lamps provision system for rural communities

George William Hong*, Naoya Abe

International Development Engineering Department (IDE), Tokyo Institute of Technology, 2-12-1-I4-4, Ookayama, Meguro-ku, Tokyo 152-8550, Japan

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ABSTRACT

Providing clean and sustainable energy for all is an ever elusive challenge, especially encountered in remote and poor rural areas. Cost-effective solutions have been found through renewable energy systems (RES) which, when combined with specialized products like rechargeable lamps using light emitting diodes (LED), can provide the basic energy needs (lighting) of rural homes, while replacing fossil-fuel based energy sources (e.g. kerosene lamps). The investigation presents an LED lamps provision system which circumvents the cost and technical challenges that currently hamper LED lamps diffusion into communities. Based on an actual rural island case (Pangan-an Island, Philippines), a sub-centralized lamp rental and charging system was mathematically modeled and analyzed (analytically and numerically) to identify the optimal states and policies, along with the effects of certain parameters, which promote financial viability and supply sustainability. It was found that a dynamically optimized lamp(s) purchase policy yields better financial returns than a statically optimized policy. Furthermore, it was realized that a sub-centralized lamps rental approach can serve as a complementary energy provision system for rural electrification projects by providing for the lower-tier energy demand market of low income users within a community.

Contents

* Corresponding author. Tel./Fax: +81 3 5734 3797.
E-mail addresses: hong.g.aa@m.titech.ac.jp (G.W. Hong), nabe@ide.titech.ac.jp (N. Abe).

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1. Introduction

To date, energy access is still a prominent challenge across the globe. The global rural electrification rate is estimated at 68%, leaving 1.3 billion people without access to electricity. About 99% of these people are found in developing countries [1]. Furthermore, most of the energy-challenged people are those with limited means and are situated in remote off-grid rural areas. Many rural households still rely on kerosene lamps to provide for the very basic needs of lighting. Kerosene lamps, however, provide inefficient light and cause indoor air pollution and hazards. In recent years, several systems such as solar home systems (SHS) incorporating light emitting diode (LED) technologies have been developed to provide the minimum household energy needs and to curb the dependence on fossil-fuel based energy sources. These technologies have recorded significant impacts to the communities: replacing conventional kerosene lamps and diesel generators, extending lighting hours for work and education, and providing long term economic benefits for communities [2]. While solar home systems are investment intensive, a variation of the system, using rechargeable LED lamps, provide an economical option for those that could not afford high investments. The widespread diffusion and usage of these LED lamps, however, has been hampered with its own technical and financial challenges. While several projects across the globe have been implemented, there is a growing need to further understand the short and long term implications of LED lamps provision system in rural communities.

This paper presents a strategic approach to rural lighting using LED lamps through a rental system which can mitigate the barriers and risks of LED lamps diffusion to communities. While focusing on the operations management and sustainability aspects, the specific objectives of the study are two-fold: a) to develop an LED lamps provision system model (sub-centralized charging and rental) and b) to identify the optimal states and policies for sustainable operations and management. The developed mathematical model is based on an actual application in an island in the Philippines (Pangan-an Island). By modeling the provision system, a better understanding of the short and long term implications of certain conditions and policies can lead to the realization of a reliable and sustainable rural lighting option.

2. LED lamps provision system

2.1. LED lamps

Solar LED lamps use LED technology, or light emitting diode, which efficiently produces light for less energy compared to other lighting technologies (e.g. incandescent bulbs and compact fluorescent light (CFL)). These LED lamps can provide about 5–12 h of light using rechargeable batteries which can be charged during day time, either through solar panel or grid electricity source. These lamps last about 1–3 years, depending on the product quality and conditions of use; which is often limited by the battery life more than the LED bulb life. The cost of good quality lamps can range from about USD15 to 30 (US dollars) while cheaper lamps can be as low as USD5. These lamps can be bought with an accompanying solar charging panel, which may cost as much as the lamps. For rural households who could neither afford to connect to the grid nor purchase individual SHSs, LED lamps can at least provide the basic needs of lighting at an affordable price. Moreover, it has been estimated that a single lamp can replace about 143 Kg of carbon dioxide (CO_2) annually by replacing 4 h of kerosene usage per day [3].

Rechargeable LED lamps have been promoted through several national and regional level projects in various countries. To list a few, in sub-Saharan Africa, the Lighting Africa program has been conducted by the International Finance Corp. (IFC-World Bank) to accelerate the development of off-grid lighting markets. It was estimated that in 5 sub-Saharan countries (Zambia, Tanzania, Kenya, Ghana, and Ethiopia), off-grid households spend about USD910 per year in both purchase and operating costs of kerosene lamps; leading to a market of about 50 Million units of modern lighting products [2]. Studies indicate that LED lamps are more efficient and economical than several home lighting systems (kerosene lamps, SHS with CFL, and diesel generator s with CFL) in the long term. Other efforts such as D.Light, Millennium Villages, and Light Up the World (LUTW) have promoted LED lamp projects to many developing countries including large markets such as China and India [3–6].

On the other hand, however, there are several challenges which hamper the widespread diffusion of these LED lamps. For one, some low-income households still could not afford the initial capital to purchase LED lamps even if these lamps are cheaper compared to full SHS systems or connecting to the grid. Another issue is the durability of LED lamps, which can become a risky investment for low-income users with less technical knowhow on the devices. Thus, in order to properly introduce and diffuse into communities, lamp projects need special funding mechanisms; either through subsidy from the government, donations, or microcredit options, which are, however, limited.

2.2. Rental and central charging

The most common way to distribute these lamps is through direct selling or the ownership model. This, however, limits sales to individuals who can afford lamps upfront. Another problem for this model is the lack of after-sales services, as lamps are sold through agents who care less about the long term function of their products. Without subsidy and strong partnership with the suppliers, diffusion of LED lamps can be very limited. One way to circumvent the challenges of direct selling has been through a rental or fee-for-service model. This allows users to use and pay for the lamps from a lamp rental operator on either a daily, a monthly, or on an as-needed basis. The rental model provides several advantages. Firstly, the daily or as-needed payment duration makes it affordable for low-income households. Secondly, a technically capable operator bares the responsibility of charging and maintaining the lamps; thus extending the lamps' service-life. There are also identified disadvantages of a rental system; users are confined to use the lamps in a limited duration and location depending on the rental conditions. Nevertheless, several projects have incorporated this approach and have hinted the effectiveness of rental and central charging [3,7]. As the burden of lamps maintenance is transferred to the operator in

a rental model, the financial condition of the operator is important to support. This ensures a sustainable supply of lamps for the overall benefit of the users. With proper consideration, the approach using a rechargeable LED lamps rental may therefore improve the diffusion and implementation of this technology into more markets. It is, however, important to further understand the implications of this approach, especially for the long term.

2.3. A lamp rental case in Pangan-an Island, Philippines

In an off-grid island (Pangan-an) in the Philippines, with 370 households, the main energy source is a solar power plant (34 kWp (kilowatt peak)) which was provided through a foreign donation (from Belgium) in year 1998. In recent years, however, the solar plant provides power only during day time, since the batteries no longer store energy for night usage. Hence, kerosene lamps and diesel generators have become a prominent source of energy at night. A recent study conducted by Hong and Abe [8,9] detailed the challenges of sustainability for the island case. The particular situation in the island allowed the opportunity to provide LED lamps charged from the solar plant during the day and to be used at night. The financial barriers of directly selling the lamps were evident in the onset, as most users had minimal financial means. The average household monthly income was only PhP4,500 (Philippine peso) while quality LED lamps would cost about PhP700—2,000 (USD 17–49) each; (PhP1 = USD41). The objective was thus to develop a lamp rental system that would require minimal amount of initial investment which would sustain the project in the long term. On September 2001, 20 units of LED lamps were initially donated to the community (PhP1,250 or USD31 per lamp) which would be available for rental [10]. The lamps were charged from the existing solar plant, rented out, and paid on a daily-basis by the users, while being overseen by the local cooperative. The profits from the rentals were used to pay for the operations cost and the purchase of additional lamps each month. Fig. 1 shows the sub-centralized lamp rental system and cycle.

As a lighting solution, the LED lamps (estimated 1.5-year service life) (30 lux at 0.5 m distance) proved to be far more effective in illuminating a room than the kerosene lamps (<0.5 lux at 0.5 m distance). The 4–6 h charging time of lamps was also appropriate for the available power from the solar plant during day time. Regarding the system economics, the daily rental rate (PhP5/lamp/day) was set to compete with the cost of using a kerosene lamp (PhP5 for 2.5 h of light). Thus, households could pay for the same amount as they used to for kerosene lamps while getting 5–9 hours of light. The monthly

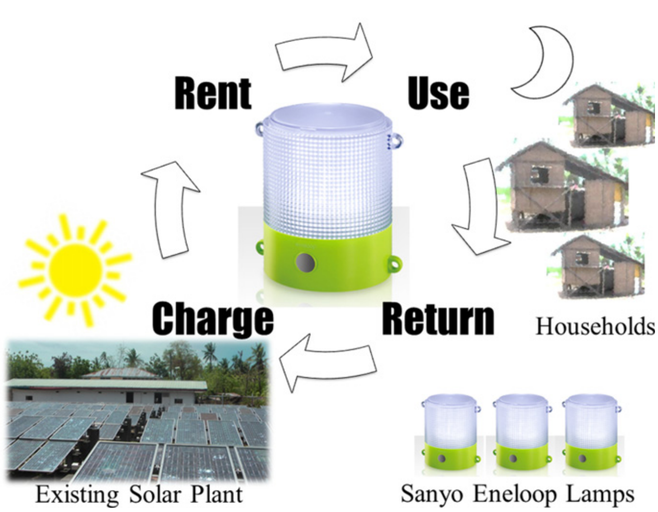

Fig. 1. Lamp rental system and cycle.

Table 1
Characteristics of LED rental lamp users (random day).

Characteristics of LED rental lamp users	Count ($n=20$)
Previous kerosene lamp users	15/20 (75%)
Fishing as livelihood	15/20 (75%)
Connected to solar grid	14/20 (70%)
Previous monthly energy expense (ave.)	PhP 167 (USD 4)
Monthly income (ave.)	PhP 3460 (USD 84)
Days willing to rent lamps (ave.)	30 days/month
Lives within 1 kM from rental station	14/15 (93%, $n=15$)

revenues of the lamps were expected to pay for the electricity consumed to charge the lamps, the fees for the rental operators, and the purchase of an additional locally sourced lamp (PhP700) per month. The rental system accommodated not only those who could not afford to buy a lamp but also those who could afford only a few days of lighting in a week; providing advantages over the direct selling method. The social acceptance of the lamps was high even at the initial stage. The rentals averaged 80% rental rate per day for the first few months of the project. It was notable that the demand for the lamps increased as users started to understand the benefits of the lamps. A daily first come first served basis was applied to have an equitable provision of lamps. As more lamps were added each month, more users could avail of the service; extending the social benefits. Further benefits were realized by the power plant operators, as the energy consumed through the lamp charging station added to the electricity sales of the existing solar plant. Table 1 shows the characteristics of the LED lamp rental users taken in a random day of operations.

This lamp provision system is expected to grow as new lamps are introduced each month. However, it may just be a matter of time when old lamps begin to deteriorate and the sustainability of the supply would be in question. The rental operators seek to identify an appropriate rental price to ensure proper financial returns. The operators also realize the importance of determining the ideal number of lamps and a replacement policy that would ensure a sustainable supply of lamps. Thus, there is merit in developing a mathematical model which can simulate and identify an optimal solution to the lamps supply scenario.

3. Modeling the rental system

3.1. Developing the general model

3.1.1. The objective function and constraints

Considering the lamp rental system in the above scenario, a mathematical model of the supply system is to be drawn. While a few lamps can be easy to manage, the service provider, or operator, needs to determine an optimal way to increase and manage the lamps supply in order to accommodate the demand in the area. Analytical and numerical analyses are to be conducted to identify and confirm the solutions to the optimal states and appropriate policies.

We begin the model development with the realization that, with a lamp rental system, the financial health of the operator is important and would evidently determine the sustainability of the supply of lamps for the benefit of the users. Thus, we postulate that the operator seeks (or is allowed) to maximize profits or the net present benefit, V, of the rental system by controlling the purchases of additional new lamps, $\{Y_t\}$ to solve Eq. (1).

$$\underset{\{Y_t\}}{\text{Maximize}}\ V=\sum_{t=0}^{\infty}\rho^t V_t=\sum_{t=0}^{\infty}\rho^t(rRX_t-cY_t) \tag{1}$$

$$X_{t+1}-X_t=-\gamma X_t+Y_t \tag{2}$$

$$Y_t \leq \frac{Z}{c}[rRX_t] \tag{3}$$

$$R = -aX_t + b \tag{4}$$

$$r = revenue_{perLamp}(rentalprice) - costs_{perLamp}(charging + personnel + maintenance) \tag{5}$$

A certain number of lamps, X_0, are initially provided by the service provider to start the rental service. The revenues from the rental per month would be used to purchase new lamps, Y_t, to add to the existing supply of lamps, X_t. Here, the net benefit or profit per period, V_t, is the difference of the gross revenue, rRX_t, less the total cost of new lamps purchased, cY_t, where c is the unit cost of a new lamp. The net revenue per lamp rented, r, is the difference of the lamp rental price per month less the costs incurred per unit lamp; which is composed of the charging cost, personnel cost and other maintenance costs, as shown in Eq. (5). We assume r to be constant through time. The rental-ratio, R, is the ratio of the number of lamps-rented per month from a total number of lamps available, X_t. This R varies depending on some parameters (e.g. supply and demand) and can have a value which ranges from 0 to 1. Multiplying R and X_t determines the number of lamps-rented per period. In order to compute for the net present value of net benefits, V, across time, we introduce a discount factor, ρ, which is defined by the equation $\rho^t = 1/(1+\delta)^t$, where δ is the discount rate. To distinguish the partial net benefits per period, V_t, and the total net present benefits through time, we will use V_{NPV} to refer to the latter. The flow equation, Eq. (2), equates the net increase of number of lamps per period, X_{t+1}–X_t, to the difference between the number of lamps that retire per period, -γX_t, and the number of new lamps added, Y_t. The number of lamps which retire or become out-of-order per period is determined by the retirement rate, γ, which can range from a value of 0–1. We assume γ to be constant. With Y_t being the control variable, we add a constraint to the values Y_t can take by limiting the cost of purchasing new lamps to the gross revenue collected per period, as seen in Eq. (3). We use a purchase policy factor, Z (0–1), to determine a proportion of the gross revenue per period which can be used to purchase new lamps and the rest to be kept as profit. This Z can be statically or dynamically set as a matter of policy.

Other necessary information can be computed from the model variables and outputs. The total energy consumption for lamps charging can be estimated from the per unit lamp consumption per charging period. Using Eq. (6), the total energy consumed per period, E_t, can be computed by multiplying the consumption per lamp, e, by the total number of lamps-rented per period, RX_t. The return of investment, ROI, can be computed from Eq. (7). The initial investment is limited to the cost of initial lamps given by cX_0. The payback period, PP, is the month when the accumulated net present benefits becomes greater than or equal to the initial investment. The abovementioned information determine the system's overall technical feasibility and financial viability.

$$E_t = eRX_t \tag{6}$$

$$R.O.I = \frac{V_{NPV} - initial\ investment}{initial\ investment} \tag{7}$$

3.1.2. The lamps-rented behavior

The lamps-rented per period is determined by the product of the total lamps in supply, X_t, and the rental ratio, R. We result to an empirical estimation of R for this model. Suppose that the system serves a limited demand of a fixed number of households. In an ideal scenario, as long as the demand is greater than the supply, all the lamps available for rental in the supply would be rented out per period. As depicted in Fig. 2, the ideal lamps-rented would increase linearly as the lamps in supply increase and then remains constant when the *demand limit* is reached. For this behavior, the ideal rental-ratio is seen to remain constant and then decreases smoothly as the *demand limit* is reached. In reality, however, the lamps-rented would be affected by two factors, the demand and supply, which may not always follow the ideal behavior. The actual demand can be less than the number of lamps in supply. The supply, on the other hand, may experience inefficiencies (e.g. lamp charging, storage, and maintenance) and thus not all lamps in supply can always be prepared to be rentable. Therefore, in an actual scenario, we can expect that the lamps-rented curve would fall below the ideal lamps-rented lines. Suppose we estimate a rental-ratio which decreases linearly. This estimated rental-ratio translates to a concave curve for the estimated lamps-rented, as seen in Fig. 2. The estimated lamps-rented curve fittingly falls below the ideal lamps-rented and follows our empirical assumptions. The peak of this concave curve is limited either by a *demand limit* or a *supply limit*. The *supply limit* is intrinsic to the operator's capacity to efficiently provide rentable lamps from the given supply. After the operator's capacity is reached, we suppose that the number of rentable lamps decreases. The *supply limit* and *demand limit* need not coincide. However, as a matter of empirical curve estimation, we assume that the peak of the estimated lamps-rented curve is less than or equal to the ideal *demand limit*. Eq. (4) depicts the rental-ratio line, where a is the slope and b is the intercept which would be empirically calibrated to the given scenario.

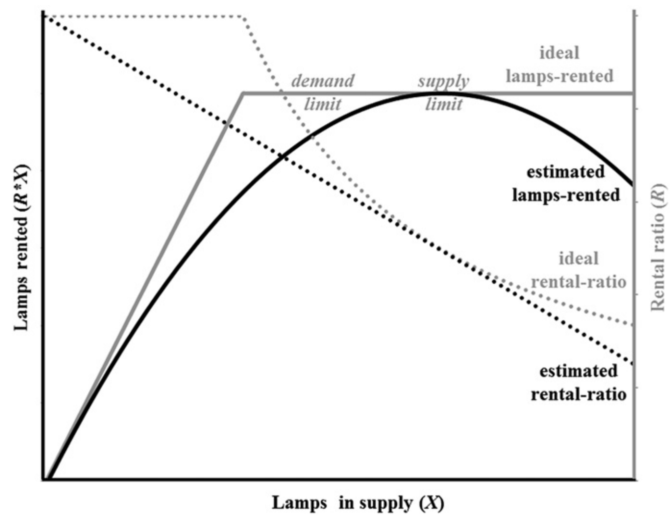

Fig. 2. Ideal and estimated rental behavior.

3.2. Static policy

3.2.1. Static purchase policy and optimal states

Having the general model at hand, we can now attempt to simulate and optimize a lamp rental supply scenario across time. As a matter of policy, one simple way to control the purchases of new lamps, Y_t, would be to fix the purchase policy factor, Z, across time. According to a given Z, the operator will allot a portion of the rental revenues to purchase new lamps while the rest is kept as profit. For this case, Eq. (3) will have an equality sign shown by Eq. (8). Although Y_t varies across time, this policy can be said to be a static optimization case since the purchase policy, Z, which is the decision variable for this case, is set from a single time period $t = 0$ and remains constant through time. As the supply changes through time there are some theoretical scenarios that can be expected which point to an optimal solution. We first identify the

analytical solution for this static model.

$$Y_t = \frac{Z}{c}[rRX_t] \tag{8}$$

Given the estimated rental behavior in Fig. 2, we can expect that there will be a certain X_{MSY} that returns a maximum number of lamps-rented, which is located at the peak of the curve. At the same peak, we can also expect that the gross revenue is a maximum and correspondingly the Y_{MSY} would also be a maximum. The X_{MSY} is called a stock level that supports *maximum sustainable yield*. We can solve for this state by using Eq. (8) and taking $\partial Y_{MSY}/\partial X_{MSY}=0$.

While we can determine the maximum output the supply can attain for a single period, a state that can be sustained into the future is even more important to determine. This state is referred to as the *steady state* where we can expect the supply level to remain constant through time. For this to happen, there should be an equilibrium between the lamps that retire and the new lamps added to the system; given by $-\gamma X_{SS}=Y_{SS}$. We can solve for the *steady state* using Eq. (2) and (8), and the relationship $X_{t+1}=X_t=X_{SS}$. This *steady state* would serve as an ideal future scenario for the system.

The analytical solutions for *maximum sustainable yield* and *steady state* presented above are applicable for a continuous state of variables. However, lamps, particularly variables X and Y, are discrete objects and thus need to be discretized. When solving for solutions to the model in discrete state, the *maximum sustainable yield* and *steady state* values will have a range of possible values due to the rounding off of discrete values. A numerical analysis using computer simulation and optimization program can be made to identify the optimal states and policy.

3.3. Dynamic policy

3.3.1. Dynamic purchase policy

Another manner of managing the lamp supply is through a dynamic policy setting. In a dynamic setting, the purchase policy varies across time to accommodate for certain optimizing criteria. In our case, the net present value, V_{NPV}, is optimized by using the optimal purchase policy which will be found through a dynamic optimization process. We first identify the analytical solution to the supply model and then discuss the numerical solution to solve the optimal states.

We, again, use the general model equations, Eqs. (1–4). For the dynamic setting, we allow the purchase of new lamps, Y, to have values that range from 0 to a maximum affordable in each period, $0 \le Y_t \le Y_{tmax}$. Thus we use Eq. (3), which is conditional, rather than Eq. (8) to control our Y. In solving for our optimal solutions we adopt the Lagrangian function of our model equations, as seen in Eq. (9).

$$L = \sum_{t=0}^{\infty} \rho^t \left\{ (rRX_t - cY_t) + \rho\ \lambda_{t+1}\left[(1-\gamma)X_t + Y_t - X_{t+1}\right] + \theta\left(\frac{Z}{c} rRX_t - Y_r\right)\right\} \tag{9}$$

Lagrange multiplier is a technique used to solve optimization problems which can be extended to dynamic conditions [11]. The first order necessary conditions are solved by setting the first order partial derivatives to 0: $\partial L/\partial Y_t=0$, $\partial L/\partial X_t=0$, $\partial L/\partial \rho\lambda_{t+1}=0$, $\partial L/\partial\theta_t \ge 0$, and $\partial L/\partial g(X, Y) \ge 0$, where $g(X, Y)=ZrRX_t/\ c\text{-}Y_t$. Through further mathematical operations, we can solve a set of values which points to a *steady state* of the system which we will term as *economic optimum*, having X^*_{EO}, Y^*_{EO}, and λ^*_{EO}. The *economic optimum* can be reached using an optimal path, where in some cases can be through a *most rapid approach path* or *MRAP* in finite time. Spence and Starret [12] described this path as the most rapid path to take X_t to X^*_{EO} by optimally choosing Y_t from its feasible set. At the *economic optimum* we can expect the following conditions which indicates a *steady state*: $X_t=X_{t+1}=X^*_{EO}$, $Y_t=Y_{t+1}=Y^*_{EO}$, $\lambda_t=\lambda_{t+1}=\lambda^*_{EO}$, and $\theta_t=\theta_{t+1}=\theta^*_{EO}=0$. For the Lagrangian function, the λ_t relates to the value of an additional unit of lamp in time t and $\rho\lambda_{t+1}$ is the discounted shadow price of an additional unit in period $t+1$. The θ on the other hand is an additional multiplier to accommodate the Kuhn-Tucker inequality of the constraint in Y from Eq. (3). This Lagrangian approach also relates to the Hamiltonian function which embodies the key concepts behind the optimal control theory and maximum principle [11,13]. Though mathematically intensive, the analytical solutions discussed above point to the dynamically optimal states and policies for the lamp supply system.

3.3.2. Dynamic programming

In a more practical approach, dynamic programming can be used to simulate and optimize our scenarios. This numerical approach uses the Bellman equation and the *principle of optimality* to efficiently search for the optimal path [11,14]. We use Eq. (10) as the Bellman equation for our model.

$$V(X_t) \equiv \underset{\{Y_t\}}{\text{Maximum}}\ [(rRX_t - cY_t) + \rho V(X_{t+1})] \tag{10}$$

This equation shows that the value function maximizes the current state with the maximum value functions of the succeeding states. Again we adopt the flow equation in Eq. (2), the constraint in Y in Eq. (3), and the rental ratio function in Eq. (4). The Bellman equation postulates that the partial derivatives with respect to Y in the value function equates to zero. The *economic optimum* points, which are equivalent to the ones found using the Lagrangian method, can be computed through numerical analysis. The X and Y variables are discretized; thus, making it a more “correct” solution for the discrete states of our lamps. The numerical analysis for this deterministic model can be compared to the analytical solutions derived from previous discussions.

3.3.3. Dynamic policy with stochastic effects

While a deterministic model mimics an ideal situation, a stochastic consideration to the supply behavior per period may allow an actual perspective to the problem. In this case, we will imply that the lamps retirement rate, γ, will vary probabilistically per period. We represent the random retirement rate by ε. The stochastic flow equation will thus be expressed as Eq. (11).

$$X_{t+1} = (1-\varepsilon)X_t + Y_t \tag{11}$$

The analytical solution to this scenario would be similar to a non-stochastic deterministic case while using the mean of the ε random component. The numerical analysis would apply an X_{t+1} transition probability range of values dictated by the characteristics of the random retirement rate. Each supply path would be randomly unique though is still expected to follow an optimal path.

4. Applying the model to the Pangan-an island scenario

4.1. Applying a static purchase policy

4.1.1. Static policy scenario

Using the lamp rental model with a static purchase policy, the Pangan-an island case can be simulated and optimized. The scenario parameters to be adopted in the model are shown in Table 2. The monthly net revenue per lamp, r, is based on a PhP6/day/lamp rental price which the operators intend to use. The calibration of the rental-ratio, R, is estimated in consultation with local experts. The *demand limit* is expected to be over 100 users while the estimated *supply limit*, as a matter of operator's capacity and efficiency to provide lamps constrained by capacity for

Table 2
Pangan-an Island scenario parameters.

Variable	Description	Value	Unit
t	Scale of period	–	Month
X_0	Initial lamps	20	Lamps
r	Monthly net revenue per lamp	150	PhP
c	Unit cost of new lamp	700	PhP
γ	Lamp retirement rate	0.05	Lamps/month
δ	Discount factor	0.05	–
ρ	Discount rate	0.952	–
a	Slope of R	0.0025	–
b	Intercept of R	1	
e	Per lamp charging consumption	24	Wh/day
–	Station charging capacity	100	Lamps
–	Initial lamps investment	14,000	PhP
–	Simulation period	120	Months

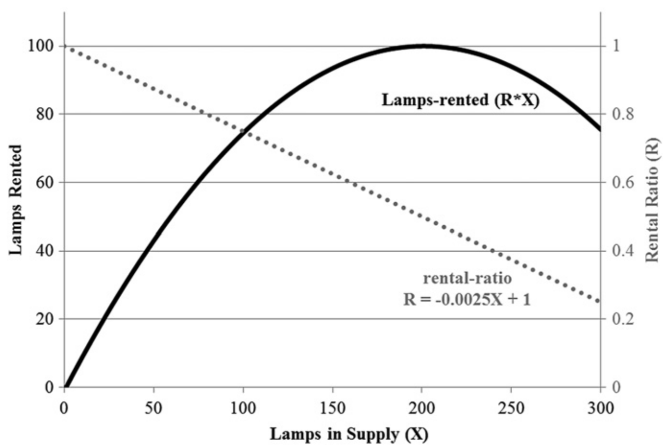

Fig. 3. Rental-ratio and lamps-rented curve for Pangan-an Island.

charging, is about 100 lamps with an efficiency of 0.5 at the *supply limit*. A slope of 0.0025 for R is found to estimate the scenario, as shown in Fig. 3. Initially, 20 lamps are provided with an assumed retirement rate of 0.05 lamps per month. Regarding the energy requirements, it was computed from actual data that the rented lamps consume an average of $e=24$ Whs (watt-hours) of energy in an average 6 h daily charging cycle. It is desired to simulate a monthly period for 120 months duration (10 years).

Using a fixed purchase policy, Z, the net present benefit, V_{NPV}, for the total duration can be solved by recursively solving Eqs. (1,2,4,8) for each period. This was done by creating a simulation program using MATLAB software. The variables Y and X were discretized by rounding the variables toward zero for each new calculation while the lamps that retire per period, $-\gamma X_t$, were rounded up. The optimal Z^*, which returns the maximum V^*_{NPV} for the objective function, can be found by a search process using different values of Z.

4.1.2. Static policy optimization results

The results of the discretized simulation and optimization process are shown in Fig. 4. The supply paths for selected values of Z are shown in Fig. 4(a). The optimal purchase policy was found to be at $Z^*=0.704$ which has a *steady state* supply $X^*_{SS}=254$ lamps. The number of new lamps purchased per period can be seen in Fig. 4(b); where $Y^*_{SS}=13$ lamps. This optimal Z^* was found by iteratively searching for the maximum net present benefit, $V^*_{NPV}=50{,}979$ PhP, as seen in the peak of the value curve in Fig. 4(c). The optimal Z^* achieved *steady state* at month 77 with an average net benefit per month of $V^*_{SS}=4{,}806$ PhP, as seen in Fig. 4(d). The *maximum sustainable yield* for optimal Z^* occurred at month 49 with $X^*_{MSY}=200$ and $Y^*_{MSY}=15$. The maximum net benefit per period occurred at month 53 with $V^*_{53}=5{,}091$ PhP. At optimal *steady state*, about 93 out of 254 lamps were rented. The rental efficiency was 36.6% which caters close to the full demand of 100 users. Using Eq. (6), the energy consumption for 93 lamps, with an average $e=24$ Whs consumption per lamp, would be 2.23 kWh (kilowatt-hours) of energy per day. This is well below the capacity of the existing solar plant which is estimated to produce 30 kWh per day. The ROI^* was computed to be 264% with a payback period, $PP^*=11$ months.

The results show that at an appropriate static policy of Z, the supply approaches *steady state*. It is observable that Z values higher than the optimum Z^* translate to more lamps in supply but return lesser net present benefits. For Z values less than the optimum Z^*, the lamps supply either remained constant ($Z=0.4$ and $Z=0.6$) or decreased to zero ($Z=0.2$). The analytical solutions were found to match the numerical results with some consideration to the discretization process. As a matter of policy setting, a fixed Z is simple for operators to follow as it uses only a fixed ratio from the budget available. However, merely following the static policy neglects the occurrence of maximum net benefit points and does not consider the supply efficiency tradeoff which may mean lesser profits.

4.2. Applying a dynamic purchase policy

4.2.1. Dynamic policy scenario

We now apply a dynamic purchase policy which will allow the purchases of new lamps to vary from $0 \le Y_t \le Y_{tmax}$. We imply a purchase policy, $Z=1$, to indicate the maximum range of the purchase allowable per period for Y. We adopt the same scenario parameters in the island case; shown in Table 2. The Bellman equation in Eq. (10) is iteratively solved via dynamic programming by discretizing the state and action variables X and Y. A computer program was developed using MATLAB software to simulate and optimize the scenarios. Components from a publicly accessible CompEcon toolbox, as presented by Miranda and Fackler [15], were utilized.

4.2.2. Dynamic policy optimization results

The results for the dynamic purchase policy scenario optimization are seen in Fig. 5. Starting from an initial number of $X_0=20$ lamps, the optimal path showed a steep incline in accumulation of lamps for supply. The *steady state economic optimum* point was reached at $t=17$ with $X^*_{EO}=100$ lamps in supply and $Y^*_{EO}=5$ lamps; as seen in Fig. 5(a,b). The optimal path accumulated a net present value $V^*_{NPV}=82{,}519$ PhP for 120 months with an *economic optimum* $V^*_{EO}=7{,}750$ PhP per month, as seen in Fig. 5(c). As with the static case, the dynamic analytical solutions matched the numerical results with some consideration to the discretization process.

The dynamic optimization process determines the optimal purchase of Y for a particular X which leads to economic optimum, as seen in Fig. 5(d). The general trend for the optimal policy of Y is to purchase the maximum lamps affordable per period until the *economic optimum* point is reached. We may term this stage as the supply accumulation stage which incorporates a *most rapid approach* or *MRAP*. If the lamps supply happens to exceed *economic optimum*, a lower range of lamps are purchased to reduce the supply back to optimal level. We may term this as the supply adjustment stage. We observe that during the supply accumulation stage, there are points prior to *economic optimum* where Y is not a maximum per period (e.g $X=56$ and $X=77$). These inflections are attributable to the discrete nature of lamps. Some supply levels are more preferred due to lesser losses in the

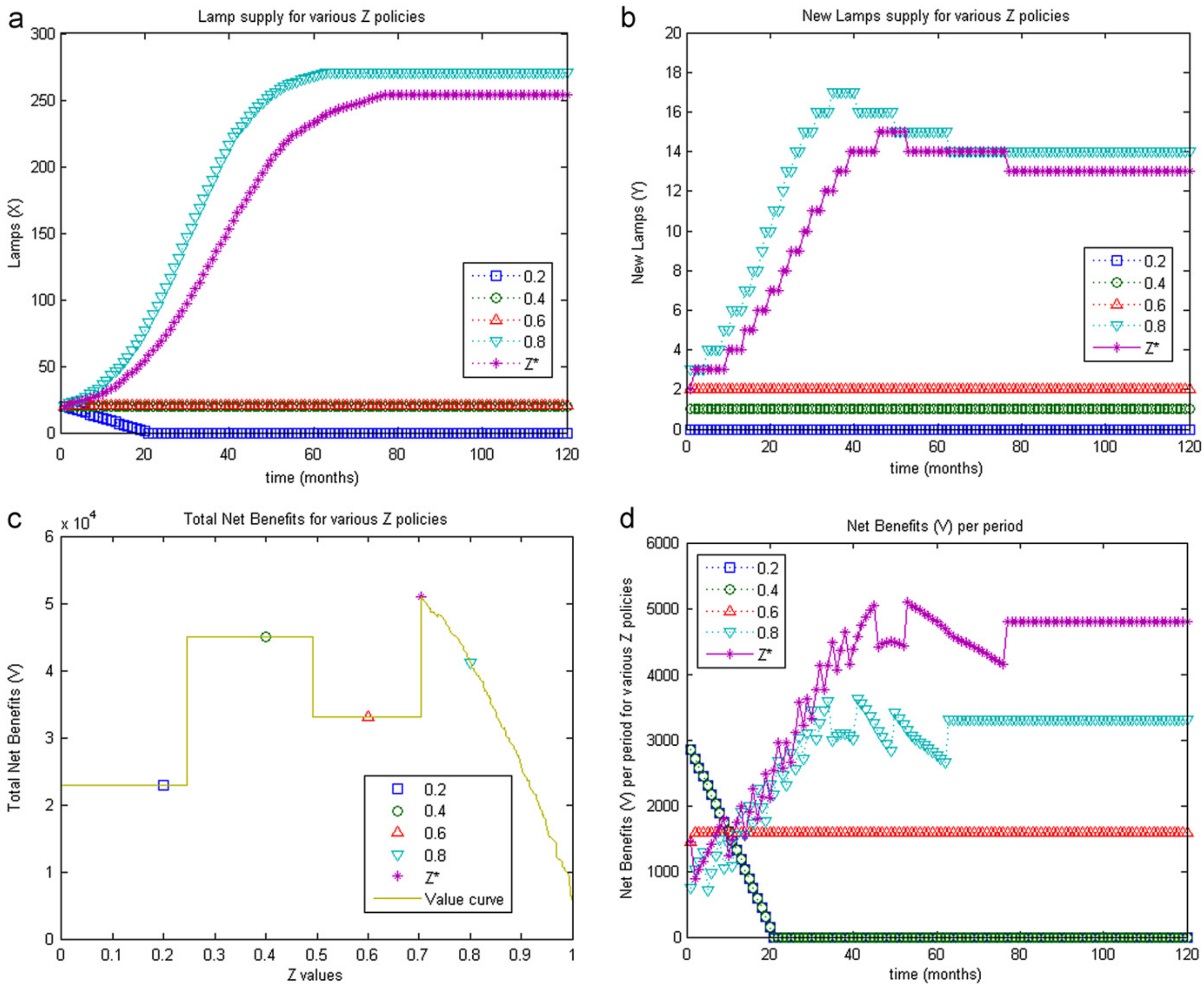

Fig. 4. Static policy simulation results for Pangan-an Island scenario.

lamps retirement which are rounded up during discretization. For example, $\gamma X^{80}=0.05*80=4$ lamps retired is preferred to $\gamma X^{81}=0.05*81=4.1$ which is rounded up to 5 lamps retired; losing a theoretical 0.9 lamps value. Thus it is found numerically optimal to adjust the previous period's purchase to achieve a supply of X^{80} rather than X^{81} in the next period. The same behavior was observed for supply values slightly higher than the X^*_{EO}. Analytically considering that these inflection points are caused by discretization without deviating from a *most rapid approach* or *MRAP* solution, we may characterize the solution to be a *bang–bang solution* between the allowable upper bound and lower bound values of *Y* at a given period. The optimal dynamic purchase policy can thus be represented by Eq. (12).

$$Y^*_t=\left\{\begin{matrix} Y^{\max}_t & if\ X_{t-1}\le X^*_{EO} \\ Y^{\min}_t & if\ X_{t-1}> X^*_{EO}\end{matrix}\right\} \tag{12}$$

At *economic optimum*, about 75 out of 100 lamps were rented. The rental efficiency was at 75% which caters to 75% of the estimated demand of 100 users. Using Eq. (6), the energy consumption for 75 lamps, with an average $e=24$ Whs consumption per lamp, would be 1.80 kWh of energy per day. This is well within the capacity of the supply from the solar power plant. The *ROI** was computed to be 489% with a payback period, *PP**16 months.

4.2.3. Dynamic optimization with stochastic effects

Using the same policy and scenario settings as the dynamic case, we introduce a stochastic retirement rate, ε, into the flow equation using Eq. (11). The random retirement rate is set with a mean of $\varepsilon=0.05$, with possible values of 0, 0.03, 0.05, 0.07, and 0.1, and with probabilities 0.05, 0.25, 0.4, 0.25, and 0.05 respectively. The scenario was simulated 1000 times with durations of 120 months.

The results of the dynamic stochastic scenario simulations are shown in Fig. 6. The stochastic retirement rate caused a range of possible values in the supply path. In Eq. (6), the stochastic paths can be found; where at the *steady state*, the range of values were found to have a minimum of $X_{\varepsilon SS(min)}=93$, a maximum of $X_{\varepsilon SS(max)}=103$, and a mean of $X_{\varepsilon SS(mean)}=98$. The stochastic *steady state* mean falls slightly below the non-stochastic steady state of $X_{SS}=100$. The mean stochastic net present benefits also falls below the discrete steady state net present benefits; where $V^*_{\varepsilon NPV(mean)}=81{,}230$ PhP, $V^*_{\varepsilon NPV(min)}=69{,}146$ PhP, and $V^*_{\varepsilon NPV(max)}=93{,}099$ PhP. This indicates that although the dynamic policy

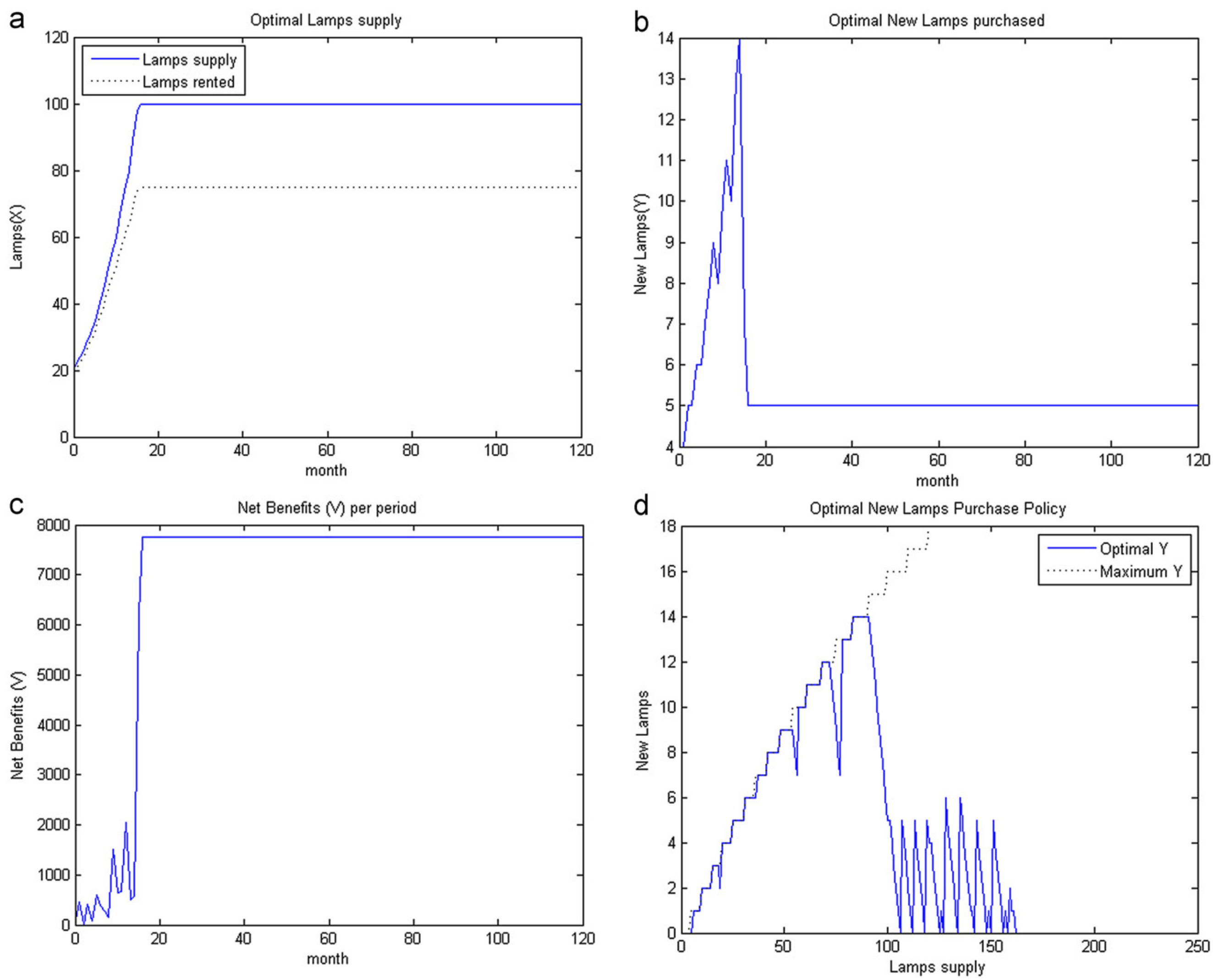

Fig. 5. Dynamic policy simulation results for Pangan-an island scenario.

adapts to the stochastic effects, on average, the supply operates slightly lower than the ideal *steady state* supply derived from the non-stochastic setting. It is found that the optimal purchase policy does not overcompensate for the possibility of higher retirement rates (stochastic) in a particular period. Correspondingly, lesser profits are expected for the stochastic supply path compared to the non-stochastic supply path. Nonetheless, the stochastic simulations resulted in an optimal *MRAP* solution similar to the non-stochastic scenario as seen in Fig. 6(b). Once again, the optimal dynamic policy equation in Eq. (12) can be used to depict the optimal path even with stochastic effects.

To further understand the effects of the range of stochastic values, further scenario testing was performed by increasing the probable range of retirement rates. Fig. 7(a) shows the result where a 1% probability of 40% retirement rate was allowed. For this case, the random retirement rate was set with possible values of 0, 0.03, 0.05, 0.07, 0.1, and 0.4, with probabilities 0.05, 0.25, 0.4, 0.25, 0.04, and 0.01 respectively; with a mean of $\varepsilon=0.053$. It was observed that the steady state supply decreased to a minimum of $X_{\varepsilon SS(min)}=42$ and returned lower financial returns. The retirement rate range was further increased by allowing a 1% probability of 80% retirement rate. The results are shown in Fig. 7(b), where the random retirement rate was set with possible values of 0, 0.03, 0.05, 0.07, 0.1, and 0.8, with probabilities 0.05, 0.25, 0.4, 0.25, 0.04, and 0.01 respectively, with a mean of $\varepsilon=0.057$. It was observed that the steady state supply reached $X_{\varepsilon SS(min)}=0$ for some simulation trials. Thus, at this retirement rate range, it is possible that the rental will be forced to stop due to excessive retirement of lamps. Considering stochastic effects, a larger range of possible retirement rates negatively affects the supply even if the probabilities of high retirement rates (80%) are low (1%).

4.3. Comparing the static and dynamic policies

4.3.1. Optimal states and policies

The comparative optimization results of the static and dynamic policies are shown in Table 3. Considering the V^{*}_{NPV}, the dynamic policy yields 61.2% more profit than the static policy for the island scenario. This is also supported by higher profits per period during the *steady state* in dynamic policy compared to static policy. The profit difference can be attributed to the number of lamps in supply achieved and maintained by the two policies. The static policy allowed the purchase of 254% more lamps compared to dynamic settings. The higher lamp supply for the static policy translated to lower rental ratio efficiency and higher required new lamps per period to maintain *steady state*. Having more lamps operated at a lower efficiency resulted to lower the profits for the static policy. The dynamic policy was found to return higher *ROI** for the investment but yielded a longer

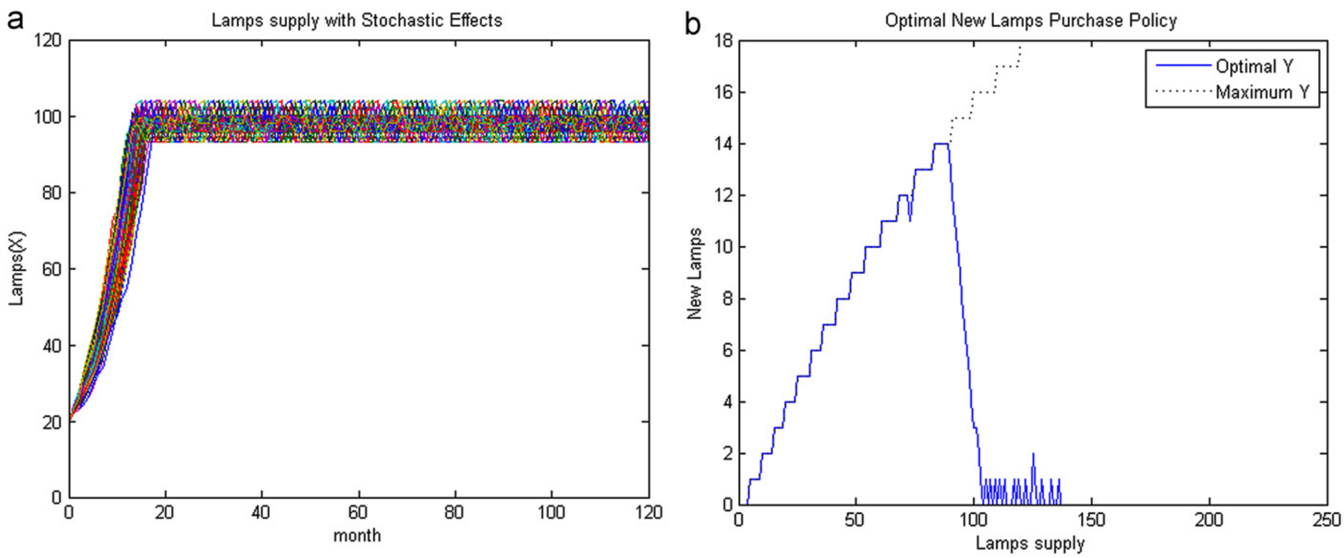

Fig. 6. Dynamic stochastic scenario simulation results.

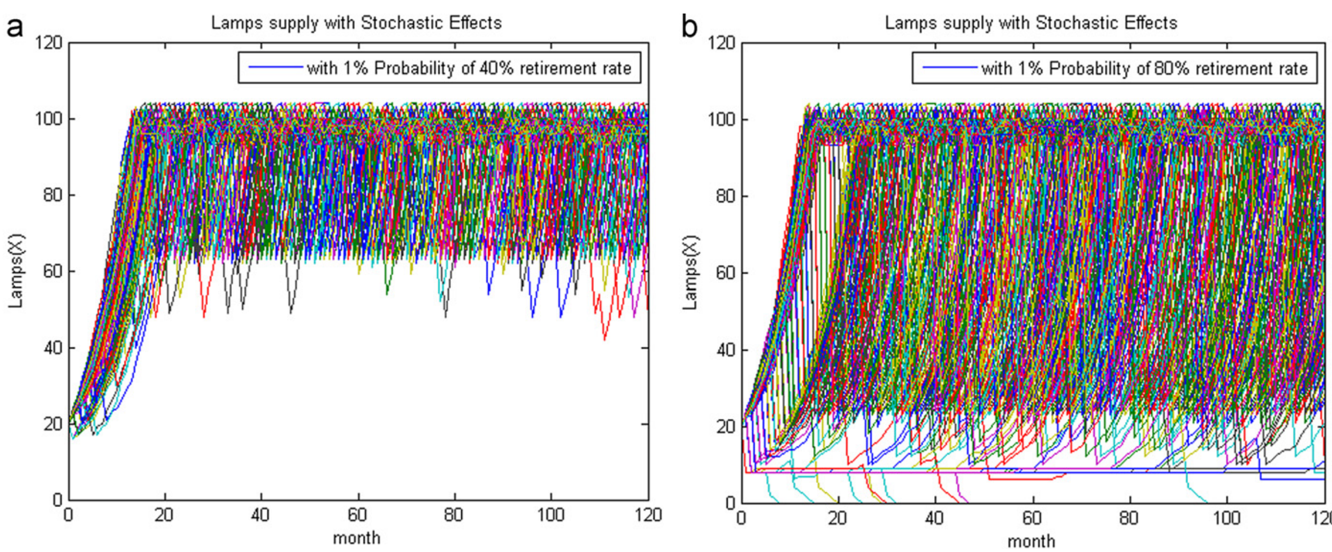

Fig. 7. Dynamic stochastic scenario: 1% probability of 40% and 80% retirement rate.

Table 3
Comparative optimal states of policies.

Optimal states	Static	Dynamic	Unit
V^*_{NPV}	50,979	82,519	PhP
X^*_{SS}	254	100	lamps
Y^*_{SS}	13	5	lamps
V^*_{SS}	4,806	7,750	PhP
t^*_{SS}	77	17	months
V^*_{max}	5,091	7,750	PhP
Z^*	0.704	–	–
LR^*_{SS}	93	75	Lamps
R^*_{SS} (*Efficiency*)	36.6	75	Percent
eRX^*_{SS}	2.23	1.8	kWh/month
ROI^*	264%	489%	%
PP^*	11	16	Months

payback period, PP^* compared to the static policy. The *MRAP* solution for the dynamic settings did not prioritize a short payback period.

Although capable of being optimized in various Z values, the optimal static policy, in general, did not capture the tradeoff in rental efficiency when pursuing more lamps in supply. The dynamic policy, however, was able to optimize the efficiency tradeoff. By supplying lesser lamps at higher efficiency, higher profits and evidently higher *ROI* were acquired. Thus in a profit oriented perspective, the dynamic policy would be favorable given that the payback period is not prioritized. On the other hand, in the demand provision perspective, the static policy provides more lamps for the users. In the island scenario where the estimated demand was 100 lamps, the static policy supply provided 93% of this demand even at a low efficiency. The dynamic policy covered 75% of the demand at optimal efficiency. This tradeoff needs to be considered when selecting the proper policy. The choice between prioritizing profit or demand may be dictated by the nature of the ownership and operation of the supply. A community-based ownership and operation may prioritize supply for the users compared to maximizing profits. On the other hand, if the system is owned and operated by an investor who is not part of the community of users, there may be a conflict between the investor's and users' interests.

For the island case, the existing solar plant was capable of providing the energy requirements of the lamp supply. It is, however, noted that a higher number of lamps for the static policy requires more energy from the solar plant compared to a dynamic policy. In either case, the energy consumption required by the rental operations contributed to additional energy sales for the existing power plant. Prior to the lamp rental operations, the low income households could not afford connecting to the power plant, which translates to a market loss for the solar plant providers. The lamp rental system was able to convert the untapped market into profit by aggregating the low energy requirements of each household. The sub-centralized lamp supply can thus be used as a complementary energy distribution system for rural electrification systems (e.g. solar plants) by providing the minimum energy needs (light) of the lower-tier energy demand market of low income users; while the main energy source can provide electricity for the upper-tier energy demand market of higher income users.

4.3.2. Sensitivity analysis

Sensitivity analysis was conducted for selected key parameters in the model to test the effects of varying scenario values to the resulting optimal states. By using the scenario values in Table 2 as the base-case scenario and varying a specific parameter (lower and higher than the base-case scenario) in both static and dynamic settings, we were able to compute and observe the tendencies of the resulting *steady state* values. A uniform 25% change in the parameters was used to allow comparison among the tested parameters; parameters were changed independently for each test with all other parameters held equal. The results are shown in Table 4 (actual values) and Fig. 8 (% change).

With regards to the change in retirement rate, lowering the γ showed an increase in V^*_{NPV}, X^*_{SS} and ROI^* while decreasing the PP^* for both static and dynamic policies; positive % changes in the outputs were greater for static policy. The opposite trend was observed when γ was increased where negative % changes in the outputs were greater for dynamic policy. For the static policy, higher Z^* was adopted to compensate for higher γ. The resilience of the static policy to an increase in γ can be explained by lower marginal profit losses when X^*_{SS} is reduced from a higher supply compared to a dynamic policy.

With regards to a change in unit price of new lamps, decreasing the c increased the V^*_{NPV}, X^*_{SS} and ROI^* while decreasing the PP^* for both policies. The opposite was observed when c was increased. For the static policy, lower Z^* was allowed with a lower c. However, the increased c affected the static policy drastically such that the supply was not able to increase from the initial state.

With regards to a change in rental revenue per lamp, lowering r decreased the V^*_{NPV}, X^*_{SS} and ROI^* for both policies. The supply level for the static policy was not able to increase from the initial state and the dynamic PP^* increased dramatically. The opposite trend was found when r was increased; for the static policy, a lower Z^* translated to higher V^*_{NPV}. Among all variable changes, a change in r had the most significant impact on the V^*_{NPV}.

With regards to a change in the number of initial lamps, increasing X_0 increased V^*_{NPV} for both policies. The X^*_{SS} decreased along with Z^* for the static policy as X_0 was increased. For the dynamic policy, X^*_{SS} remained unchanged regardless of the change in X_0. Compared to other variable changes, a change in X_0 had the least effect in % change of outputs. In all test scenarios with a higher $X_{0(15,\ 20,\ 25,\ 50,\ 200)}$, the behavior for the static policy was not desirable since the lamps supply reduced as profit was maximized. When lamps are abundant initially, the necessity to purchase new lamps yields to the premium of earning income with less expense in the early periods. Thus, in a static policy, the initial lamps strongly influence the *steady state* supply.

Since the retirement rate of lamps is quite related to the price of lamps, a special case analysis was conducted to investigate the tradeoff between the two parameters. Two scenarios were analyzed by simultaneously changing c and γ: (I) expensive and more durable lamps (c increased by 25% as γ decreased by 25%) and (II) cheaper and less durable lamps (c decreased by 25% as γ increased by 25%); all other parameters had base-case values. The results of the special case analysis are shown in Table 4. It was found that cheaper and less durable lamps returned higher profits and *ROI* for both static and dynamic cases. Thus, the lamp quality and cost tradeoff, with proper consideration, can be exploited by the operator as a strategy to improve financial returns.

Table 4
Sensitivity analysis results: actual values of X^*_{SS}, V^*_{NPV}, ROI^* and PP^*.

Parameters	Static policy					Dynamic policy			
γ	Z^*	X^*_{SS} (lamps)	V^*_{NPV} (PhP)	ROI^* (%)	PP^* (months)	X^*_{SS} (lamps)	V^*_{NPV} (PhP)	ROI^*(%)	PP^*(months)
0.0375	0.557	256	65,313	367%	8	106	98,119	601%	15
0.05	0.704	254	50,979	264%	11	100	82,519	489%	16
0.0625	0.737	225	45,151	223%	12	96	68,416	389%	18
c									
525	0.557	261	76,182	626%	7	120	117,490	1019%	12
700	0.704	254	50,979	264%	11	100	82,519	489%	16
875	0.308	20	41,362	136%	9	80	55,201	215%	21
r									
112.5	0.328	20	30,105	115%	10	80	36,167	158%	24
150	0.704	254	50,979	264%	11	100	82,519	489%	16
187.5	0.563	254	88,279	531%	7	120	137,330	881%	13
X_0									
15	0.704	254	44,554	324%	10	100	72,806	593%	19
20	0.704	254	50,979	264%	11	100	82,519	489%	16
25	0.601	221	66,301	279%	10	100	90,493	417%	15
50	0.459	141	105,850	202%	9	100	120,270	244%	11
200	0.275	60	208,930	49%	13	100	221,830	58%	11
Special case analysis I (c=875 and γ=0.0375) and **II** (c=525 and γ=0.0625)									
I	0.696	256	49,023	180%	12	106	71,683	310%	21
II	0.553	225	70,220	569%	7	128	103,690	888%	14

Initial investment cost for lamps (PhP700/lamp): $X_{0(15)}$=10,500; $X_{0(20)}$=14,000; $X_{0(25)}$=17,500; $X_{0(50)}$=35,000; and $X_{0(200)}$=140,000

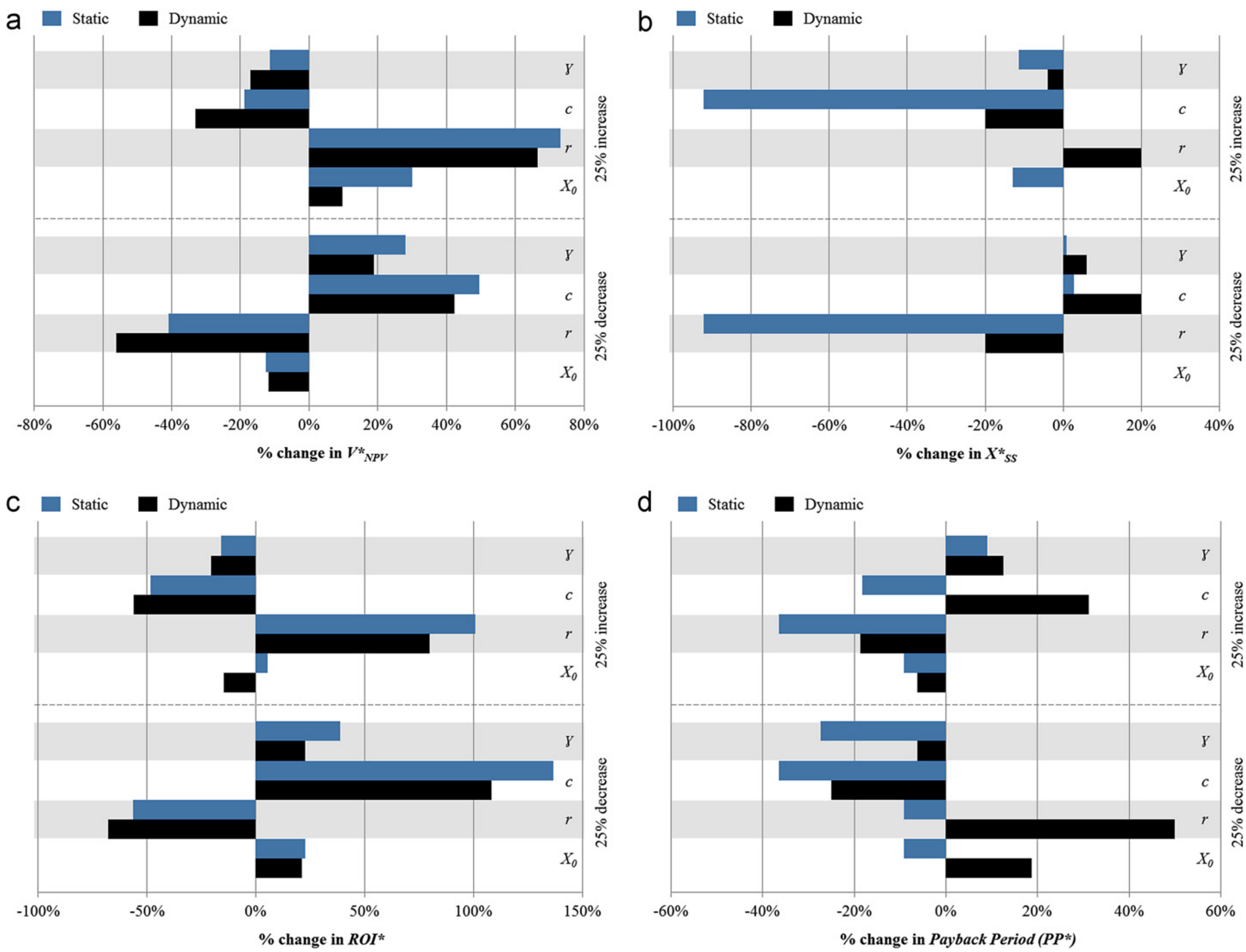

Fig. 8. Sensitivity analysis results: % change in V^*_{NPV}, X^*_{SS}, ROI^* and PP^*.

Considering the impacts of all parameters, r had the most significant impact on the V^*_{NPV}. A 25% increase in r had 73% and 66% increase in profits for the static and dynamic cases respectively. Increasing r and decreasing c and γ had positive impacts on the V^*_{NPV} and ROI^* The level of significance of parameters to the change in V^*_{NPV}, whether positive or negative, is ranked as follows: r, c, γ and X_0; where X_0 is least significant. However, an increased X_0 for the static policy was found to have a decreased X^*_{SS} which can be considered a negative compromise to the users. It was also found that the static policy was more sensitive when an increase in V^*_{NPV} and ROI^* was expected while the dynamic policy was more sensitive when a decrease in V^*_{NPV} and ROI^* was expected. In an investment perspective, for all cases with varying $X_{0\ (15,\ 20,\ 25,\ 50,\ 200)}$, ROI^* was always positive with a PP^* of at most 24 months (2 years).

To improve the financial viability of the system, increasing r would be the most significant strategy which does not compromise supply levels. The affordability of the rental price must of course be considered. Since c was found to be more significant than γ, another feasible strategy would be to search for lower priced lamps while sacrificing a bit of quality with higher γ. For the static policy, the Z value can be set high to prioritize profit or set low to prioritize supply. Further, identifying the proper range of X_0 is important to determine the proper *steady state* supply. For the static policy, too many initial lamps, X_0, would lead to a low *steady state* supply and would compromise financial returns; specifically, *ROI*. As for the dynamic policy, which is flexible with a wide range of X_0, an appropriate X_0 can be identified to have the optimal economic returns without compromising the *steady state* supply.

4.3.3. *Policy feasibility*

An important feature for a sound policy would be the feasibility of its implementation. While the dynamic policy yielded more profits, it may not necessarily be the easiest policy to operate with. Given the scenario where the operators in the island have limited capacity to assess the supply and demand for optimal settings, it would be difficult to decide the optimal purchases on a monthly basis. Moreover, the actual occurrences would deviate from the simulated scenarios using the model based on empirical assumptions. None the less, references similar to the *optimal purchase policy graph* in Fig. 5(d) and the dynamic purchase policy equation in Eq. (12) can be used to identify the pre-calculated optimal purchases relating to the supply level per period. A static policy, on the other hand, simply applies a fixed budget ratio for the operators to follow for the succeeding months. A steady growth of lamps can be expected until the *steady state* point, which adjusts to the profits received per period, is achieved.

In selecting the proper policy, there should be consideration in more aspects of the project than simply the supply financials and apparent supply level outputs. According to the World Bank [16], there are three important design principles for renewable energy projects in developing countries: getting the prices, the institutions, and the context right. With that in mind, a rental system should adopt a policy which considers the following: affordability of users and viability of investment, the proper participation of the stakeholders, and the overall need and dynamics (process of provision) of the lamp supply system.

5. Conclusions and recommendations

With the ongoing search for sustainable solutions to improve energy access in hard to reach areas, renewable energy systems incorporating rechargeable LED lamps have been increasingly promoted. The cost-effectiveness of these lamps has been noted in several projects and studies. The widespread and sustainable diffusion of these lamps have, however, been hampered due to technical and financial barriers. The perceived solution, as depicted in this investigation, is through a rental system using a sub-centralized charging station which operates on several merits: (1) a daily or as-needed rental is more affordable and circumvents high initial investment for low income users, (2) trained operators can charge and maintain the lamps better, and (3) the rental system can augment the energy sales of rural power suppliers by aggregating the low energy requirements of a possibly untapped market of low income households.

For the off-grid rural island case presented, an LED lamp rental system was successfully implemented to provide benefits for low income users. The rental operators, however, lacked a long term perspective of the operations management of the supply system. In order to understand the dynamics and sustainability of such rental supply systems, a mathematical optimization model was developed. Two feasible policies were identified: (1) a static policy which uses an optimally fixed ratio for the profit and the budget to purchase new lamps, and (2) a dynamic policy which flexibly identifies the optimal number of new lamps in a per-period basis. The analytical solutions of the model were identified to lead towards optimum *steady states* in the long term. Numerical analyses were conducted for both policy settings which simulated the optimal *steady state* supply and the financial returns for the given conditions.

Through the numerical optimization process, it was identified that the dynamic policy yielded better financial returns (61.2% higher profits) than the static policy. The dynamic policy applied a *most rapid approach* to reach *steady states* which maximized profits, but yielded longer payback periods. The static policy tended to have higher *steady state* supply, but operated at less efficient levels. Thus, in a profit oriented perspective, the dynamic policy is preferred; while in the demand provision perspective, the static policy may provide more lamps for users. The ownership of the system may dictate whether supply or profit is prioritized; a community-based ownership may prioritize supply while an outside-investor ownership may prioritize profits. Stochastic effects (using random retirement rates) were applied for the dynamic setting and yielded a probabilistic range of values which can be expected in the actual scenario. The stochastic scenario on the average tended to have lower supply than the ideal optimal steady state supply level and thus returned lower profits. The effects of variations in model parameters were also confirmed. The parameter of the net rental revenue per lamp was found to be most significant in influencing the financial returns; a 25% increase in net revenue per lamp had 73% and 66% increase in profits for the static and dynamic cases respectively. The cost per unit lamp was found to affect the financial returns more significantly than the lamp retirement rate. Thus, allowing a cost-effective tradeoff between the price and quality of lamps. A high number of initial lamps was found to negatively influence the *steady supply* achieved in a static policy setting, while the dynamic policy was more flexible to a wider range of initial number of lamps. In all scenarios tested, the rental system achieved positive return of investments, payback periods of at most two years (for the lamps), and energy requirements well within the energy source capacity. Thus, the investigation confirmed the long term financial viability and technical feasibility of the LED lamps rental system for the rural island scenario.

The feasibility of applying the policies, however, needs further consideration. While the dynamic policy setting yields better profits, a static policy setting may be easier to apply for rural conditions; operators can be expected to have limited experience and understanding of the lamp rental market. Thus, a proper policy and supply scenario setting should consider the financial aspects, the stakeholders involved, and the dynamics (process of provision) of the supply system. Nonetheless, a good understanding of the on-site scenario would provide the best basis for policy setting. While the developed mathematical model was based on an actual case, the theoretical considerations may reduce the model's ability to simulate actual occurrences. It is recommended to improve the model by confirming results using long term in-situ data while incorporating other possible important parameters.

Overall, the investigation yielded several useful insights for the operations management, policy selection, and conditions setting for LED lamps rental systems for rural communities. The technical feasibility and financial viability of a rental provision system has been confirmed along with the identification of key parameters and variables which influence sustainability. Further, it was realized that a sub-centralized lamp rental system can provide benefits as a complementary energy provision system for rural electrification projects by providing for the lower-tier energy demand market of low income users within a community. The outputs established in this investigation are intended for project developers, investors, manufacturers, and prospective users who wish to adopt this alternative energy provision system and bring sustainable light into homes and communities where energy is rarely an everyday commodity.

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